NPS-OR-04-002

NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

Optimization-Based Military Capital Planning

by

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October 2004

Approved for public release; distribution is unlimited.

Prepared for: Air Force Office of Scientific Research
(Optimization and Discrete Mathematics)
4015 Wilson Blvd.
Arlington, VA 22203-1954

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REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*

1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** October 2004	**3. REPORT TYPE AND DATES COVERED** Technical Report

4. TITLE AND SUBTITLE: Optimization-Based Military Capital Planning	**5. FUNDING NUMBERS** ROR9G
6. AUTHOR(S) Gerald G. Brown, Robert F. Dell, and Alexandra M. Newman	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000	**8. PERFORMING ORGANIZATION REPORT NUMBER** NPS-OR-04-002
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) Air Force Office of Scientific Research, (Optimistic and Discrete Mathematics), 4015 Wilson Blvd., Arlington, VA 22203-1954	**10. SPONSORING / MONITORING AGENCY REPORT NUMBER**

11. SUPPLEMENTARY NOTES The views expressed in this technical report are those of the authors and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.	**12b. DISTRIBUTION CODE**

13. ABSTRACT *(maximum 200 words)*

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14. SUBJECT TERMS Resource planning, capital planning, capital budgeting, military planning, integer programming, model tractability, the art of modeling	**15. NUMBER OF PAGES** 44
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	**18. SECURITY CLASSIFICATION OF THIS PAGE**	**19. SECURITY CLASSIFICATION OF ABSTRACT**	**20. LIMITATION OF ABSTRACT**
Unclassified	Unclassified	Unclassified	UL

Optimization-Based Military Capital Planning

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ABSTRACT

The United States military carefully plans and justifies its materiel procurements. These decisions have a profound, long-term impact on our ability to defend our nation, and to fight and win our nation's wars. Annual U.S. materiel investment is now larger than that of the rest of the world combined, and attracts keen attention from political leaders and government contractors. Procurement plans are complicated by their influence on domestic technology and production abilities, conflicted objectives, concerns regarding interoperability and maintainability of the materiel, and the sheer scale of the endeavor. Mathematical optimization models have long played a key role in unraveling the complexities of capital planning, and the military has lead the development and use of such models. We survey the history of optimizing civilian and military capital plans and then present prototypic models exhibiting features that render these models useful for real-world decision support.

Keywords: Resource planning, capital planning, capital budgeting, military planning, integer programming, model tractability, the art of modeling.

1. What's so Complicated about Capital Planning?

Procurement of materiel has been a concern of the American military since the Revolution. Most early procurement requests amounted to not much more than a field officer's handwritten letter for "what we must have to accomplish this task." With the exception of unique military demands for critical goods such as saltpeter, most supply requests were simple—of the sort civilians would buy—and were for modest quantities. Debate centered on how to pay for what was required, rather than whether the need was real. (To gain an appreciation of the predominant role military procurement played in Revolutionary times, we recommend the writings, annotated diaries, and biographies of those who debated and decided these issues, *e.g.*, McCullough's *John Adams* [2001].)

Planning U.S. military procurement remained relatively short-term, reasonably simple, and motivated by apparent need—but driven by immediate affordability—until after World War II. In 1948, the Hoover Commission required that the military set forth its defense goals and the means by which it would achieve these. In the early 1960s, Secretary of Defense Robert McNamara tried to mitigate the myopia of a single-year budget plan and accurately represent defense systems too complex to be procured and fielded in a single year. He adopted a five-year budget requiring analytical justification. This foundation underlies our military planning today: each branch of the military forms a strategy, categorizes this into "mission areas," and translates these into requirements for personnel and materiel. For a detailed history of military funding and its consequences, see Chambers [1999].

Despite early simplicity in justifying military expenditures, U.S. military capital planning has always involved large amounts of resources from many parts of the country, extensive research effort and technological development, significant amounts of money, and the attention of political leaders. In 1794, the USS Constitution (Figure 1) and her five sister frigates—costing $800,000 (1794 dollars, or about $2.9 billion 2003 dollars [Field 1999], or more than $1,000 2003 dollars per capita)—were approved for construction using the newest technology and resources from all the colonies, on the condition that the ships be built exactly as proposed in six different American constituencies. The B1B Lancer, a $300 million bomber with an unprecedented combination of payload, speed, and range, entered service in 1986 after assembly in Palmdale, California, from components originating in all 50 states.

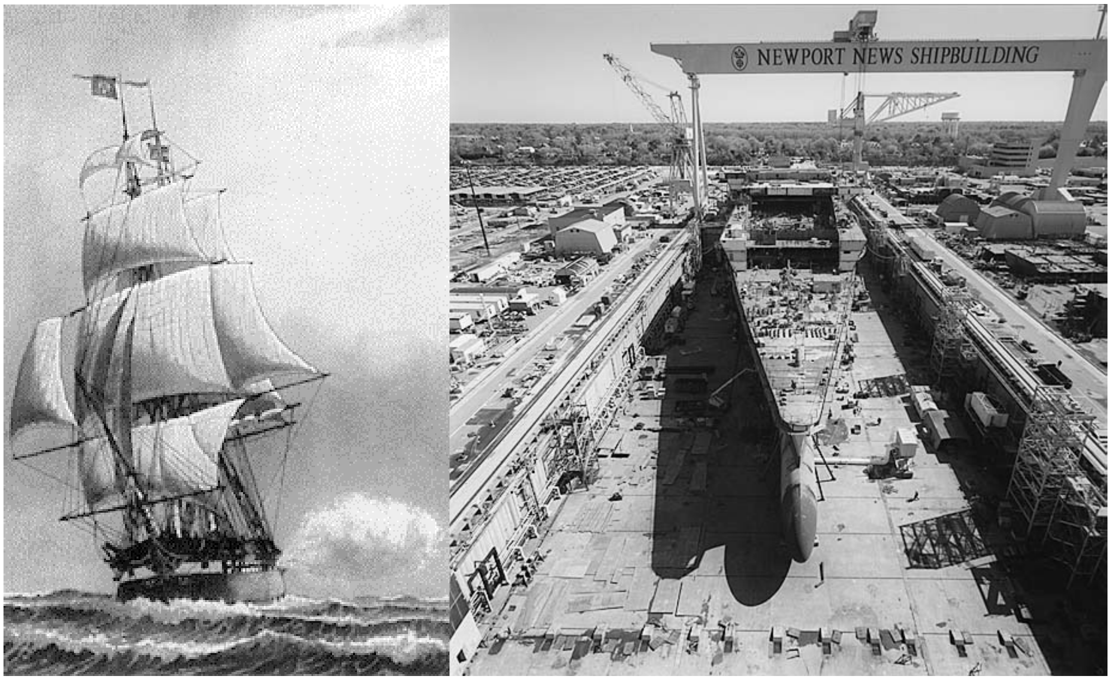

Figure 1: The USS Constitution incorporated innovative naval architecture and the latest armament technology; the highest levels of American government planned and approved its construction in 1794, and it required a huge mobilization of colonial resources.

Newport News Shipbuilding, the sole shipyard in the United States capable of building nuclear-powered aircraft carriers, builds the USS Ronald Reagan (CVN76). The cost for the Reagan and her aircraft is about $10 billion 2004 dollars.

Modern procurement planning may involve programs that require many years to develop and complete. Requirements and costs may change during program development. For example, the Army had to reprogram “Star Wars” research in 1985, responding to a $100 million cost overrun [Reuters 1985]. About six years later, the Navy cancelled development of an attack jet after unanticipated program costs increased by billions, drawing criticism of long-term Navy acquisition planning [Pasztor 1990, 1991]. In 1999, the Navy suffered a $100 million cost overrun in fielding the Joint Strike Fighter (Figure 2) [Ricks 1999].

Figure 2: The Lockheed Martin X35 Joint Strike Fighter has a planned unit cost of about $40 million, with production ramping up for 500 planes over fiscal years 2005-2010, deliveries starting in 2008, initial operational capability in 2011, and a total planned production campaign of 3,000 aircraft. The X35 will replace the U.S. Air Force's A-10 and F-16, the U.S. Marine Corps' AV-8B and F/A-18, and the U.S. Navy's F/A-18. The three services' long-term capital plans must reflect the influence of this transition across all these aircraft and their weapon systems.

Because capital planning is an important, complex, and expensive problem, it invites careful analysis. Since the introduction of mathematical programming after World War II, the military, as well as the private sector, has used it to solve capital planning problems, and the resulting decisions have committed trillions of dollars. We distill from an extensive literature some important contributions to optimizing capital budgets, distinguishing between the military and the private sector. We present prototypic models exhibiting key features that make these models useful for real-world decision support, and discuss what we frequently need to do to enhance the fidelity and responsiveness of these models. This material does not appear in any single reference, and some of it has never appeared at all in the open literature.

2. Military Capital Planning Versus Civilian Capital Budgeting

2.1 Optimizing Military Capital Planning

Some of the earliest papers in the journals of the new discipline of operations research address military capital planning. Bailey [1953] presents what we would today call a systems engineering analysis of the costs and benefits of alternate bomber designs. Stanley, Honig, and Gainen [1954], in their timeless paper, offer an insightful analysis of the real-world complexities of bid evaluation to determine the "true minimum cost to the Government," and include optimization modeling advice. Marschak and Mickey [1954] show how to select a weapon system based on its fixed cost in order to maximize expected military utility using a convex nonlinear program, and invoke the then-new conditions of Kuhn and Tucker [1950] to characterize optimal solutions.

Buffum [1978] addresses a ubiquitous military problem: given a fixed budget and set of binary alternatives, each with a cost and a priority, how does one choose the best portfolio? In this case, the alternatives are Navy research test flight packages, and costs include those of several types of limited resources, *e.g*., manpower, hangar space, and flight hours. Buffum employs linear integer programming to prove, with an objective bound on solution quality, that he has an optimal portfolio. Pfarrer [2000] studies a problem similar to Buffum's: an optimal shopping list for special operations weapons. He solves the problem manually, and then offers both a fast heuristic and a linear integer program for certifying performance of the heuristic.

Taylor, Keown, and Greenwood [1983] analyze military aircraft procurement, seeking to minimize violation of respective elastic constraints on peacetime and wartime objectives, conformation of aircraft components, combat ability, cost limits, payload and range, maintenance, accidental losses, and flexibility (interoperability) with other allied systems.

Brown, Clemence, Teufert, and Wood [1991] develop a large-scale linear integer model for modernizing the Army's helicopter fleet over a multidecade planning horizon. The model determines when and how many of each type of potential or existing aircraft to manufacture new, to extend the life of, to improve, and/or to retire. The objective accounts for costs of procurement, operations, and maintenance, subject to limits on average fleet performance and age, expenditures, production and manufacturing, and upgrading aircraft. Their model offers a range of alternate production campaigns for each type of helicopter, start date, stop date, and production rate, thus capturing volume discounts and learning effects with new technology. Loerch, Koury, and Maxwell [1999] employ an optimization model to recommend a mix of Army weapon systems that maximizes force effectiveness, subject to budget, production, force structure, and logical constraints on allowable mixes of systems. The Army used the results in both of these cases.

Brown, Coulter, and Washburn [1994] describe a nonlinear optimization model used for more than 20 years to recommend Air Force purchases of conventional gravity bombs. The single-period model assigns sorties in a war theater subject to weapons availability, aircraft sortie rates and effectiveness, target numbers, vulnerabilities, values, and weather states. The myopic model is used over a many-period planning horizon with transitions between periods allowing for target regeneration, reinforcements, war phase changes, *etc*. Yost [1996] addresses the same problem and contributes an omniscient linear model of the entire planning horizon of the war theater, with refinements such as a conditional probability model of the weather state at the target given the weather forecast at takeoff. These real-world models have been used for a long time.

Newman *et al*. [2000] and Brown, Dell, Holtz, and Newman [2003] describe a linear integer model the Air Force has used to select and schedule investments of space-based assets over a 25-year horizon. The model recommends a mix of research and development projects, current systems, and launches that minimizes shortfalls in task performance, while adhering to constraints on budget, launcher demand, launcher availability, and logic governing the precedence and interdependence of systems operating in any given epoch.

Field [1999] presents a linear integer model to plan Navy force structure over a 25-year horizon, scheduling production and retirement of various types of ships, while adhering to constraints on spending, keeping shipyard workloads stable, and maintaining required force levels. Baran [2000] enhances Field's model with additional ship classes, aircraft types, and funding categories. Garcia [2001] adds even more detail, accounting for aircraft age, and age-dependent operation and maintenance costs.

Bruggeman [2003] introduces a decision support system for suggesting procurement plans for about 40 categories of conventional Navy ordnance over an eight-year planning horizon. His goal is to find budget-feasible procurement plans that take advantage of quantity discounts, keep key production lines active, and build inventories synchronously toward a military requirement. His purpose-built heuristic produces solutions very quickly for display in a graphical user interface, but an integer linear program is used for important cases.

Baker, Morton, Williams, and Rosenthal [2002] develop a model for the Air Force Studies and Analyses Agency to show how an entire Air Force strategic airlift can optimally incorporate cargo aircraft, refueling aircraft, airport availability, ground handling equipment, *etc*. In particular, they show how to evaluate the marginal contribution of new cargo aircraft. The Agency used this model as a decision support tool, *inter alia*, to help make fleet-mix decisions, and to suggest the best use of aircraft that can both act as aerial refuelers and haul cargo.

Some of these military capital-planning models express how the assets would be used, rather than how they should be procured. Brown, Coulter, and Washburn [1994] do not address capital planning at all; they don't even have a budget constraint. However, they do show how any particular stockpile of conventional bombs would be used with available platforms to fight an emergent theater war. Their model is as much normative as prescriptive, and the key is melding data from a myriad of sources into a unified model of a complex air war. The result is used to justify munitions capital plans. Similarly, Baker, Morton, Williams, and Rosenthal [2002] have no budget constraint or costs, but the model they describe can be used to, for instance, determine what effect introducing a new cargo aircraft has on the ability to airlift materiel to various remote locales. The model is complicated, but the result is a simple quantitative assessment of return on such an investment.

2.2 Optimizing Civilian Capital Budgets

While there is extensive literature on optimizing civilian capital budgets, we find few reports of actual applications, real-world solutions, and corresponding insights. We highlight the applications we do find within our select history of capital budgeting optimization models.

Reports of optimizing capital budgets appear as early as the 1950s. Gunther [1955] presents, in the form of an amusing story to disguise the client, an industrial capital budgeting linear program with six continuous variables. A few early papers illustrate how interest rates affect spending. For example, Lorie and Savage [1955] seek to maximize company net worth. Their manual methods yield near-optimal solutions for most of their small, numerical examples spanning multiple time periods and admitting dependencies between candidate investments. Kaplan [1966] suggests a generalized Lagrange multiplier method to solve the problem posed by Lorie and Savage. Kaplan notes that his method can produce many solutions, rather than just one, by violating a budget constraint that may not need to be strictly observed.

A well-written survey by Weingartner [1966] presents a series of multiple time period capital budgeting models. His deterministic integer and nonlinear models capture dependencies among investments, and he suggests probabilistic models for addressing uncertainty. His review includes solution methods fashionable at the time, such as dynamic programming and Lagrangian relaxation, and he offers some computer code.

Bernhard [1969] gives a review of capital budgeting literature by presenting a general prototypic formulation. He analyzes his linear model with Karush-Kuhn-Tucker conditions, and then demonstrates how published models are a special case of his formulation. He suggests, but does not heartily recommend, that chance constraints and expected values can be used to express uncertainty. Weingartner [1977] reviews literature on "capital rationing" models, and criticizes published advice on determining discount rates and specifying an objective function.

Integer programs of useful size remained beyond the capabilities of commercial software for some time, but linear programs were solvable. Baumol and Quandt [1965] examine a simple capital budgeting model that maximizes net discounted cash flow from project construction subject to a simple budget constraint for each time period. They then generalize by allowing funds to carry over from one period to the next, and suggest a way to use the budget constraint dual from a slightly modified model with an objective of maximizing utility from consumption (rather than maximizing the net discounted cash flow from investments) to obtain a reasonable discount rate. Carleton [1969] reconciles Weingartner with Baumol and Quandt by suggesting a restatement of the objective to express "macro" decisions of a firm (*e.g.*, return on stockholder equity), rather than the "micro" consequences of these decisions (*e.g.*, investment and return money streams). Bhaskar [1974] extends Weingartner to account for borrowing and lending. Lusztig and Schwab [1968] apply sensitivity analysis to assess the discount rate of the cost of investment candidates that would make them attractive in an optimal solution.

Their multiple-time period models exhibit time-dependent costs between candidates, but not dependencies between them. Bradley and Frey [1978] analyze a multiple-time period linear program with time-varying costs by investment type to illustrate how the dual multipliers of budget constraints can be used to deduce favorable discount rates. Ashton and Atkins [1979] claim that rules of thumb solve a simple multiple-period linear program fairly well, with profit contributions varying by investment candidate and year, subject to cash flow constraints and simple upper bounds on the amount of investment in each candidate. They offer a 30-candidate, 26-year example.

Later, integer programs became more tractable, both via improved enumeration software and thanks to heuristics that provide relatively good solutions to special classes of integer programs. Senju and Toyoda [1968], and Toyoda [1975] address one such class, the binary knapsack problem with multiple constraints (an optimal portfolio selection problem). Each paper introduces an "effective gradient measure" to gauge the incremental effect of each investment (or divestment) based on cost and consumption of limited resources. The former method would today be called a greedy, myopic, feasibility-seeking deletion heuristic, and the latter an addition heuristic. Each paper demonstrates by example that a near-optimal solution can be quickly achieved for a relatively large problem (at that time), with a thousand binary alternatives. Dobson [1982] develops bounds on solution quality for these heuristics, and, thereby, conditions on parameters for which these heuristics are dependable or not.

Unger [1970] formulates a model similar to Baumol and Quandt [1965] and suggests solving it with Benders Decomposition. He also suggests implicit enumeration, in lieu of linear programming-based enumeration, to solve the integer linear master problems. Keown and Martin [1976] present a single-period linear integer model to select a set of investments for a hospital. They formulate a goal program to motivate, *e.g.*, spending of earmarked funds, staying within a restricted budget, and purchasing a required subset of systems. A small example illustrates their model. Rychel [1977] presents a multiple-time period linear integer program to maximize net worth for Cities Service Company, while accounting for many realistic side constraints such as short- and long-term borrowing, and total expenditures. Costs vary by candidate investment, year, and budget level. Fox, Baker, and Bryant [1984] propose the incorporation of research and development projects into a Lorie and Savage-style capital budgeting model. They argue that these types of projects exhibit interdependencies, and develop a formulation and solution technique for such a model. Bradley [1986] maximizes short- and long-term net present value for General Telephone and Electronics Corporation subject to financial, resource, and service constraints. His linear integer program has time-varying costs by investment type, and considers dependencies among candidate investments. He shows how scrupulous model formulation, including elastic (linear) goal constraints, enables optimal solution of models with thousands of constraints and binary alternatives.

Evans and Fairbairn [1989] present a linear integer program to suggest a portfolio of NASA space missions that best meets humanistic, intellectual, and utilitarian goals, while adhering to constraints on mission compatibility and budget. These authors present an example with 24 candidate missions, and one or two alternate start years each. Kumar and Lu [1991] present a case study to plan inputs and outputs for a fertilizer plant that maximize net present value as a function of market type, subject to constraints on budget, supply and demand, capacity, and dependencies between outputs. Parametric analysis of a small five-period model examines the influence of changes in costs and demands.

Although many of these models are motivated by real-world examples, only Rychel [1977] and Bradley [1986] report actual experience and the insights that accrue from such. That is likely why these two papers exhibit such a wealth of insight.

Bradley's work is in a quintette of papers including Edwards [1986], Geoffrion [1986], Glomski [1986], and Sweet [1986] that provides a rare, high-level glimpse into the rationale, design, implementation, documentation, and use of a complete portfolio optimization decision support system to guide about $3 billion in nationwide telecommunication hardware investments. Distinctive here is the use (attributed by Bradley [2003] to Geoffrion) of operating statements—funds flow and balance sheet—to communicate among executives, spreadsheets, and optimization models. The spreadsheets offer descriptive, as well as prescriptive, optimized views of the five-year planning horizon. These statements, once calibrated with the rest of corporate operations, convey constraints on internally-raised funds, external borrowing, and ultimately return per share on common stock. Thus, Bradley's optimization model simultaneously optimizes capital portfolio outlays and the mechanics of their financing in capital markets. (We are grateful to Bradley for sharing with us his complete files on this project.)

There is also a body of academic literature focused more on solution techniques than on solutions of specific capital budgeting problems. In an insightful paper, Everett [1963] demonstrates how Lagrangian relaxation can be applied to (integer) knapsack-like problems in which the goal is to maximize an arbitrary payoff function subject to a set of resource constraints. While the procedure is not guaranteed to provide an optimal solution for every scenario, he shows that optimal solutions often result, and provides insights as to why the procedure sometimes fails to converge and how this shortcoming can be rectified. Broyles [1976] shows how to algebraically remove all continuous slack fund variables from Weingartner's model to produce a smaller, pure-integer model that might be solved manually or with then-fashionable implicit enumeration. Today, this reduction is an automatic presolve feature "remove a singleton column from its equation" of contemporary solvers. Nonetheless, we like the straightforward presentation style.

Mamer and Shogan [1987] devise a custom heuristic for choosing which parts to include in a repair kit, such that the part cost is offset by the revenue generated by performing repairs. Their single-period model exhibits costs by part and revenues by repair, but does not allow dependencies between parts and/or repairs. The authors propose problems with hundreds of alternatives and thousands of constraints, and adopt Lagrangian relaxation and a greedy heuristic to suggest feasible integer solutions. Karabakal, Lohmann, and Bean [1994] propose a linear integer equipment-replacement model to determine the types and times of equipment purchases and retirements to maximize net present value subject to constraints on budget and interoperability. The authors present an example with about a thousand binary alternatives and a hundred constraints, devise problem reductions to isolate easy Lagrangian subproblems, and suggest a multiplier adjustment heuristic. Lofti, Sarkis, and Semple [1998] present a model to determine when to invest in manufacturing systems and how to allocate the corresponding amount of production among different part types to maximize company net present value, which depends on the part and system combination employed, subject to constraints on costs and time interdependencies for operating each system. The authors use variable substitution to obtain an optimal integer solution with their linear programming relaxation, and demonstrate their method on a relatively small problem. Kimms [2001] studies project scheduling, but embellishes conventional activity precedence constraints and activity costs with period-by-period cost constraints. His model determines which projects to select, and for each selected project the start date of each activity. His objective is to maximize on-hand cash at the end of the horizon.

Kimms applies Benders Decomposition to decouple the monolithic problem into project subproblems, while preserving a master schedule of period-by-period cash accounting. His test problems exhibit about 50 time periods, and have a maximum of 50 projects and 60 activities per project. His objective converges to optimality in a matter of seconds for some instances, while large decomposition gaps (on the order of 50%) persist for others.

Finally, several authors have formally incorporated uncertainty, *e.g.*, Keown and Taylor [1980] suggest new equipment purchases for a textile manufacturer facing uncertain demand. They employ chance constraints for a single time period, while expressing capacity, demand, supporting equipment, and profit goals. They offer a 20-candidate example. Meier, Christofides, and Salkin [2001] seek to maximize the time-varying value of a portfolio of investment options. Proposing a linear integer program with a budget constraint and dependencies between candidate investments, they estimate portfolio value uncertainty from samples of real options and offer not only a heuristic solution, but also a heuristic bound on solution quality and a three-period example to illustrate.

2.3 How do Military and Civilian Capital Planning Differ?

Dantzig's [1963, Chapter 2] recollections clarify why the military rushed the development of linear programming just after World War II: "A nation's military establishment, in wartime or in peace, is a complex of economic and military activities requiring almost unbelievably careful coordination in the implementation of plans produced in its many departments." Papers edited by Koopmans [1951] provide additional history on the early use of linear programming by the U.S. military.

Military investments involve enormous amounts of money, enough that economists since the 1960s have developed a defense economics literature (*e.g.*, Sandler and Hartley [1995]). Brown, Clemence, Teufert, and Wood [1991] advised decisions committing between $35 and $100 billion over a 25-year planning horizon. Brown, Coulter and Washburn [1994] report how capital expenditures of $2 to $4 billion per year have been planned for more than 20 years. Newman *et al*. [2000] and Brown, Dell, Holtz, and Newman [2003] model investments of more than $300 billion over a 25-year planning horizon. Salmeron, Brown, Dell, and Rowe [2002] advise capital outlays of about a trillion dollars over a 25-year planning horizon. The sheer scale of these military programs dominates private-sector, big-ticket investments (*e.g.*, the $3 billion reported by Bradley [2003], or the $5 billion Iridium satellite constellation).

Government investments are encumbered by so much specific congressional legislation of how and where money must be spent that political insulation of capital investments is a paramount concern. Since the mid-1950s, defense investments have been justified by detailed intermediate-term statements of the required capability, and the relation between this and the requested expenditures: The Program Objective Memorandum (POM) is a six-year plan beyond the current fiscal year that essentially fixes spending in the early years and merely announces intent for later-year expenditures. For purposes of major capital outlays, this plan is only revised once per fiscal year, promulgated up through the Department of Defense (DoD) chain of command and then submitted from the executive branch to Congress and debated. Private investments are not customarily shackled this way.

Congressional funding of DoD programs is subject to continuous review and revision by legislators.

> Article 1, section 9, of the U.S. Constitution stipulates that "No Money shall be drawn from the Treasury, but in Consequence of Appropriations made by Law." This appropriation power, in conjunction with the more specific constitutional charges to "raise and support Armies" and "provide and maintain a Navy," gives Congress tremendous say over the budgets, structures, and duties of the armed

> forces. The Constitution forbids Congress from making defense appropriations more than two years in advance, and by custom appropriation laws are passed annually. In addition to using the appropriations power to determine how much the armed services may spend, Congress can use the appropriation power to bar the armed services from undertaking specified programs or operations. [Chambers 1999, p. 179]

To ensure that this legislation is adhered to, military investments are subject to an astonishing array of audits and investigations, arising from an alphabet soup of agencies: the Government Accounting Office (GAO), Congressional Budget Office (CBO), House and Senate Appropriations Committees (HAC and SAC), and a galaxy of nongovernmental watchdog agencies leveraging the Freedom of Information Act. Optimization has a distinguishing advantage under audit: you can state your assumptions, publish your data, and make strong assertions about the quality of the solutions you report. On one visit by HAC analysts (with military backgrounds) to an author of this paper, after going over a multitude of data sources, details, and modeling assumptions, the key question arose: "how can we be sure this answer is optimal?" Brown recalls: "Oh, be still my heart!" Karush-Kuhn-Tucker conditions were offered for consumption, and there were no follow-on questions at all.

Military capital planning models must not only account for the way planning is overseen and regulated, but must also accommodate concerns not predominant in the private sector. For example, defense programs have high research, development, test and evaluation costs, usually starting many years before a system is fielded, and sometimes greatly exceeding the per-unit production cost of the resulting system. Few private sector companies endure costs of such magnitude or duration.

Defense investments not only buy capital equipment, they nurture military-essential domestic production capability and technology. For example, the Army manages conventional ammunition production and is the sole domestic consumer of certain energetic chemicals and large-caliber gun barrels. The Army must sustain this key research and technology production base, while economically producing peacetime research and training rounds, and ensuring that the U.S. always has the reserve capacity to replenish wartime ammunition consumption quickly [Bayram 2002]. The U.S. Navy is the sole domestic consumer of high-pressure steam nuclear reactors. Defense investments may enforce monopoly (*e.g.*, government-owned, government-operated facilities) and/or monopsony (*e.g.*, the purchase of restricted military hardware).

There are other differences between the civilian and military sectors that influence the way in which models are constructed, received, and, correspondingly, used. Private-sector companies must book investments according to general accounting standards and cannot deduct the entire investment as an expense offsetting current income, but must rather depreciate the investment over time. Because private-sector companies typically raise capital by borrowing, the availability, cost, and restrictive terms of these loans have an effect on investment decisions. The private sector justifies investments with "return on investment" (ROI) analysis, or "payback time"; such outcomes are conventionally expressed as "return on owner equity," and this is an aggregate function of, *e.g.*, the present value of funds invested over time, funds returned from investments over time, the costs and encumbrances incurred by borrowing, tax exposure, and all financial interactions between discretionary investments and other operations. Many of the most esoteric mathematical arguments in the literature derive from these paradoxically-complex details in the private sector. By contrast, government capital outlays are viewed as immediate expense items from the current fiscal year budget. Although the government does borrow money and publishes a current cost of capital, there is no direct connection between the timing and cost of such borrowing and the use of the borrowed funds. Government budgets are approved and strictly administered by fiscal year. Private-sector bookkeeping may fall into fiscal years for tax and disclosure purposes, but this does not significantly restrict the timing or payment for capital purchases. DoD agencies subdivide fiscal year budgets into strict spending categories (*e.g.*, payroll versus hardware accounts). Private sector companies do not.

Our military is a meritocracy. Senior military officers have risen through the ranks via both successful operational and administrative performance. There are no shortcuts: the military has no concept of assigning a fresh MBA directly to a field officer billet. The military requires its executives to have first-hand, lower-level experience before deciding policy from a higher level. There are few such apparent requirements in the private sector, and this distinction leads to differences in the way solutions to capital planning models are viewed, presented, and accepted.

Our military is a technocracy. Modern militaries value technology as a key component of force. The ranks of uniformed and civilian military decision makers are rife with degreed scientists and engineers, many with postgraduate education and ample foundation experience applying their academic training to exigent military topics. These decision makers and their support analysts often have first-hand experience with the types of weapons systems and the issues at hand.

The cost of failure is inestimable. The military has learned that there is no substitute for scrupulous planning and the insights that accrue in the abstract, rather than in the reality of battle.

3. Optimization Models

We present prototypic models for optimizing military capital planning that illustrate some transcendent features. These models prescribe which weapon systems should be procured, when they should be procured, and how many should be procured. We restrict our discussion to deterministic models, even though many future details are uncertain. However, we make allowances for uncertainties through the usual deterministic equivalent mechanisms of safety stocks, factors that emphasize the importance of near-term obligations, and the application of a range of alternate future scenarios (*e.g.*, plans for various theater wars) to assess solution robustness.

3.1 Portfolio Selection, aka the Knapsack Model

One of the simplest optimization models for military capital budgeting is a binary knapsack. Given a fixed budget and a set of binary *acquisition options*, where each option has a value and a cost associated with the procurement of one or more weapon systems, we seek the set of options that has the maximum total value at a portfolio cost no greater than our budget.

This model is a linear integer program with a linear objective and a single linear inequality constraint with nonnegative coefficients. The binary selection requirement makes this problem potentially impossible to solve exactly in reasonable time, yet we encounter and solve shopping list problems like this every day.

For this simple model, we make standard linear programming assumptions: additive objective values and additive costs, constant returns to scale, separable options, and deterministic data. In terms of real-world acquisitions, this means that the total portfolio value is just the sum of its component values, and there is no synergism among selected options. There are no returns to scale, volume discounts, mutually exclusive or inclusive options, and we have absolutely perfect knowledge *a priori* of the exact consequences of any action we might choose.

Using methods ranging from a simple myopic heuristic to formal optimization, despite all these assumptions and limitations, we can deliver useful, real-world solutions of known quality. This simple model is a valuable decision-support tool.

In the real world of capital planning, there are important embellishments beyond the textbook binary knapsack problem. In particular, we frequently must decide whether or not to buy any of a weapon system, and then decide how many of the system to buy. We may also have several acquisition options available for procuring a weapon system. For this generalization, we present a bounded integer knapsack model, *e.g.*, Bertsimas and Tsitsiklis [1997] Chapter 6, with one "measure of effectiveness" (MOE). For guidance on developing MOEs and on scoring a system's contribution towards an MOE see, for example, Keeney [1992] and the application by Parnell *et al*. [1998]. Loerch [1999] presents a linear integer program for the instance in which contribution (or cost) decreases nonlinearly as the integer quantity of the system procured increases. (Phenomena like this arise from quantity discounts, learning curves, and diminishing returns.) We can capture this nonlinear contribution (or cost) with a piecewise linear function by using binary selection variables, each of which assumes either a value of one, if the acquisition option is chosen at the associated quantity, or zero otherwise.

An embellished knapsack model is defined as follows:

Indices and index sets:

a = acquisition option,

w = weapon system,

$w(a)$ = set of weapon system(s) procured under acquisition option a.

Parameters [units]:

l_{aw} (u_{aw}) = lower (upper) limit on quantity of weapon system $w \in w(a)$ available for purchase under acquisition option a [w units],

$fixedcontr_a$ = the fixed contribution of acquisition option a toward the MOE [value units],

$varcontr_{aw}$ = the variable contribution per unit of weapon system $w \in w(a)$ [value units/w unit],

$fixedcost_a$ = the fixed cost incurred by selecting acquisition option a [\$],

$varcost_{aw}$ = the variable cost per unit of weapon system $w \in w(a)$ purchased under acquisition option a [\$/w unit],

$budget$ = the available budget [\$].

Decision Variables:

$SELECT_a$ = 1 if any units are purchased under acquisition option a, = 0 otherwise [binary],

$QUANTITY_{aw}$ = the number of units of weapon system $w \in w(a)$ purchased under acquisition option a [w units].

The corresponding linear integer program is:

$$\underset{\substack{SELECT,\\QUANTITY}}{\text{maximize}} \sum_a \left(fixedcontr_a SELECT_a + \sum_{w \in w(a)} varcontr_{aw} QUANTITY_{aw} \right)$$

$$\text{s.t.} \sum_a \left(fixedcost_a SELECT_a + \sum_{w \in w(a)} varcost_{aw} QUANTITY_{aw} \right) \le budget$$

$$l_{aw} SELECT_a \le QUANTITY_{aw} \le u_{aw} SELECT_a, \ \forall a, w \in w(a)$$

$$SELECT_a \in \{0,1\} \ \ \forall a$$

$$QUANTITY_{aw} \in \{0,1,2,\cdots,u_{aw}\} \ \forall a, \ w \in w(a).$$

Restating the effect of the above, either $SELECT_a = 0$ and $QUANTITY_{aw} = 0$ for all w in $w(a)$, or $SELECT_a = 1$ and $l_{aw} \le QUANTITY_{aw} \le u_{aw}$ for all w in $w(a)$. If purchase quantities are sufficiently high, we can reasonably relax the integrality requirement on $QUANTITY_{aw}$. For example, Salmeron, Brown, Dell, and Rowe [2002] use integer quantities of Navy ships (for about three ships per year), but continuous Navy aircraft quantities (for about 100 aircraft per year). For simplicity, we assume hereafter that continuous quantities will suffice.

Expenditure for military assets is often restricted to a specific funding category, or "color of money" (Figure 3). Each category derives from the appropriation Total Obligation Authority (TOA) from which the funds come. For example, Navy money for aircraft is categorized as "Air Procurement, Navy," and ship money is called

"Shipbuilding and Conversion, Navy." Each category of money is associated with its own restrictions such as the time by which, and the way in which, the money must be spent; additional restrictions may include the rate at which, and the assets on which, the money can be spent. We account for these categories by adding an index for funding category *c*, and modify our budget constraint slightly:

$$\sum_{a}\left(fixed\,cost_{ac} SELECT_{a} + \sum_{w \in w(a)} var\,cost_{acw} QUANTITY_{aw} \right) \leq budget_{c} \;\forall c.$$

Even with these multiple budget constraints, heuristics such as Senju and Toyoda [1968] can be generalized to work quickly as long as the contributions and costs can be expressed with a common sign. In general, we still prefer to optimize with linear integer programming when we can afford the computational delay.

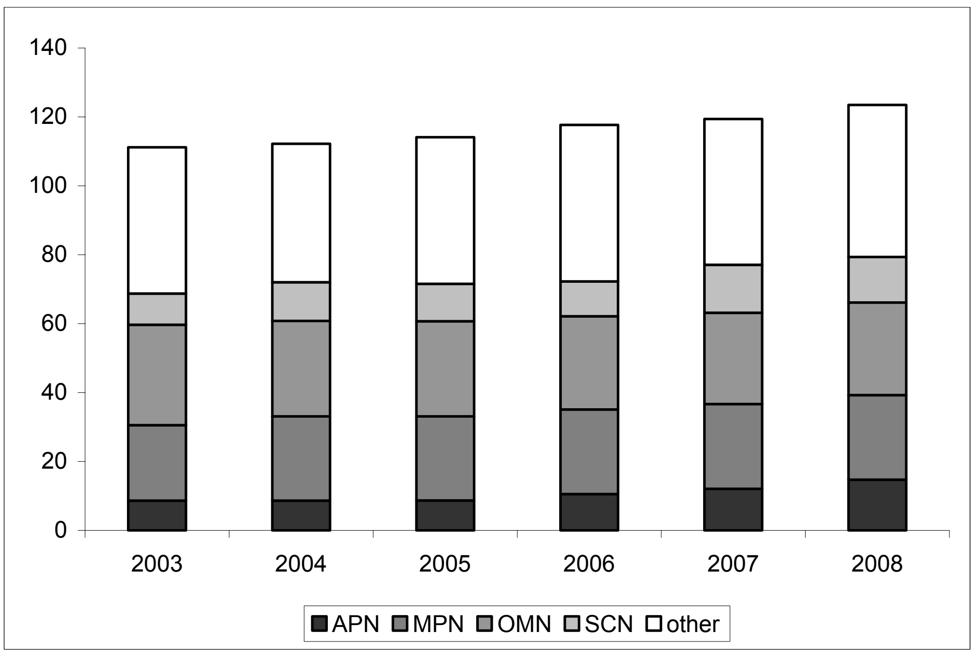

Figure 3: The U.S. Department of the Navy TOA (a grand total spending limit) extracted from the Defense Department's future years defense program (FYDP: the multiyear spending plan) as of the January 2003 submission of the president's budget for fiscal year 2004. Shown are fiscal year accounts in constant 2003 billions of dollars and five earmarked colors of money (APN is Aircraft Procurement, Navy, MPN is Military Personnel, Navy, OMN is Operation and Maintenance, Navy, SCN is Shipbuilding and Conversion, Navy, and "other" represents an aggregation of 20 additional categories). In capital-planning parlance, fiscal year 2003 is the current year, 2004 and 2005 are budget years, 2006 and beyond are out years. In this case, the 2004 Department of Navy spending is as submitted for congressional approval following reviews by the Office of the Secretary of Defense and Office of Management and Budget.

3.2 Interactions Among Decisions

Some acquisition options may require, or preclude, others. For example, there may be 10 acquisition options for a new tank (weapon system) and, at most, one may be selected. Newman *et al*. [2000] give examples such as a satellite, which, if funded, requires a capable launch vehicle. The acquisition options governing the satellite and a capable launch vehicle are otherwise completely independent.

We define a "coercion set" as a group of acquisition options that are involved in some restriction associated with the selection of each of them.

Common coercion sets include:

- "select at most, exactly, or at least k of these acquisition options";
- "this acquisition option must be selected to enable any option in that set to be selectable"; and
- "if any acquisition option in this set is selected, then at least one in that set must be selected."

These coercions arise, for example, to "keep a shipyard open," "maintain redundant sources," "exercise a contract option," or "limit the number of simultaneous selections."

The effects of weapon system contributions are varied and often interact. Given weapon system w procured under acquisition option a, and weapon system w' procured under acquisition option a', we can model pairwise interactions that do not depend on the quantity procured. We use an additional binary variable $BOTH_{aa'}$ that has value one when both a and a' are purchased, along with the following linear constraints:

$$BOTH_{aa'} \leq SELECT_a$$
$$BOTH_{aa'} \leq SELECT_{a'}$$
$$BOTH_{aa'} \geq SELECT_a + SELECT_{a'} - 1.$$

3.3 Multiple-Year Planning Horizon

Most capital planning for major weapon systems extends over the likely lifetime of the systems, but no further than we are willing to risk forecasting the future. When considering a planning horizon as long as 20 or 30 years, we usually keep track of the year in which a weapon system starts service (*start year*) and perhaps the year in which it stops service (*stop year*). The former time period can correspond to the acquisition decision year, payment year, and/or the first service year. For ease of exposition, we assume that these years coincide, although reality is more complicated, and we show later that time lags usually separate these events. We also track the weapon systems in inventory (adding new purchases and deducting retirements of old weapon systems) and we can account for operating costs that vary with the service life, or age, of each system. An acquisition option a is endowed with a specific start and stop year, as well as minimum and maximum yearly purchase quantities, for its associated weapon system(s). For multiple-year planning, converting costs to some base present value year is an inestimable convenience. For military planning, several discount rates are published, *e.g.*, Naval Center for Cost Analysis [2002].

This gives rise to a generic multiple-year model:

Indices and Index Sets:

$a =$ acquisition option,

$c =$ color of money,

$w =$ weapon system,

$y =$ year, alias $\underline{y}$,

$w(a) =$ set of weapon system(s) procured under acquisition option a.

Parameters [units]:

l_{awy} $(u_{awy}) =$ lower (upper) limit on quantity of weapon system $w \in w(a)$ purchased in year y under acquisition option a [w units],

$fixedcontr_{awy} =$ the fixed contribution of weapon system $w \in w(a)$ toward the MOE [value units] in year y under acquisition option a,

$varcontr_{awy} =$ the variable contribution per unit purchased of weapon system $w \in w(a)$ toward the MOE in year y under acquisition option a [value units/w unit],

$fixedcost_{acy} =$ the fixed cost in color of money c in year y incurred by selecting acquisition option a [\$],

$varcost_{acwy} =$ the variable cost in color of money c per unit of weapon system $w \in w(a)$ purchased under acquisition option a in year y [\$/w unit],

$budget_{cy} =$ the available budget in year y in color of money c [\$].

Decision Variables:

$SELECT_a$ = 1 if any units are purchased under acquisition option a, = 0 otherwise [binary],

$QUANTITY_{awy}$ = the number of units of weapon system $w \in w(a)$ purchased under acquisition option a that begin operation at the end of year y [w units],

$SERVICE_{wy\underline{y}}$ = the number of units of weapon system w in service during year y that first served at the end of year $\underline{y}$ [w units],

$RETIRE_{wy\underline{y}}$ = the number of units of weapon system w taken out of service at the end of year y that first served at the end of year $\underline{y}$ [w units].

The corresponding integer linear program is:

$$\underset{\substack{SELECT, QUANTITY,\\ SERVICE, RETIRE}}{\text{maximize}} \sum_{a,y,w \in w(a)} \left(fixedcontr_{awy} SELECT_a + varcontr_{awy} QUANTITY_{awy} \right)$$

$$\text{s.t.} \sum_a \left(fixedcost_{acy} SELECT_a + \sum_{w \in w(a)} varcost_{acwy} QUANTITY_{awy} \right) \le budget_{cy} \quad \forall c, y$$

$$l_{awy} SELECT_a \le QUANTITY_{awy} \le u_{awy} SELECT_a, \; \forall a, w \in w(a), y$$

$$\sum_{a | w \in w(a)} QUANTITY_{awy} = SERVICE_{w,y+1,y} \quad \forall w, y$$

$$SERVICE_{wy\underline{y}} = SERVICE_{w,y+1,\underline{y}} + RETIRE_{wy\underline{y}} \quad \forall w, y, \underline{y} < y$$

$$SELECT_a \in \{0,1\} \quad \forall a$$

$$QUANTITY_{awy} \ge 0 \; \forall a, \; w \in w(a), \; y$$

$$SERVICE_{wy\underline{y}} \ge 0 \quad \forall w, y, \underline{y}$$

$$RETIRE_{wy\underline{y}} \ge 0 \quad \forall w, y, \underline{y}.$$

When we select an acquisition option a, it may inflict fixed and variable costs over many years. The generalized $fixedcost_{acy}$ and $varcost_{acwy}$ parameters allow us to schedule both the fixed and variable costs for each acquisition option annually and make them payable over many years before and after the weapon system begins service. These cost parameters allow a fixed lag between the time a system is paid for and the time it begins service.

Costs, such as operating and maintenance costs, may vary by planning year and also by the age of the system. To this end, we can define $varcost_{acwy\underline{y}}$ as the variable cost in color of money c during year y for a weapon system w under acquisition option a given that the system is y - $\underline{y}$ years old. $SERVICE_{wy\underline{y}}$ is the number of units in service during year y that are y - $\underline{y}$ years old.

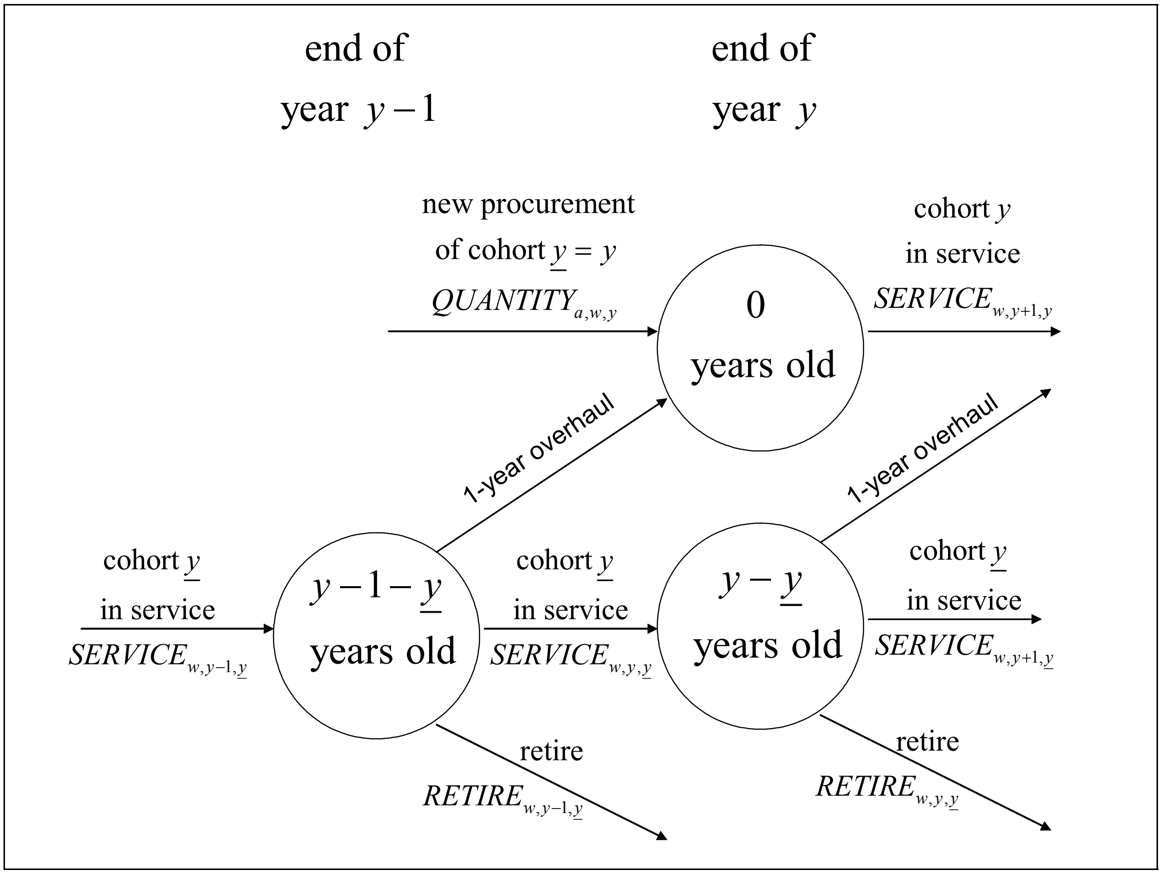

Figure 4: To track the age of an asset we must account for both its introduction (cohort) year $\underline{y}$ and its service year; each cohort is a unique commodity, and we need conservation-of-flow constraints that retain this distinction. Some service-life-extension actions can overhaul an old asset to create a new one. Inventory ageing is an essential complication when, for example, maintenance cost varies by age, or if the asset has a maximum planned service life; this can dramatically increase the size of long-term capital planning models.

Similarly, we may need to force overhaul and retirement decisions by constraining maximum service life or some burdened function of service life and usage rate.

Aged inventory is always an issue, so it is curious that textbooks rarely mention it.

We can relax the budget constraint in the above formulation by accumulating both expenditures and the budget allowance up to any current period to produce the following set of *cumulative constraints*:

$$\sum_{a}\sum_{y'\le y}\left(fixedcost_{acy'}SELECT_a + \sum_{w\in w(a)} varcost_{acwy'}QUANTITY_{awy'} \right) \le \sum_{y'\le y} budget_{cy'} \quad \forall c,y.$$

In this way, we can retain unused funds from one year to pay for an acquisition in a subsequent year with greater benefit. In reality, we may not be able to apply past funds to future years: many budgets are granted on a use-or-lose basis. However, by using a model to forecast when we need funds, we may be able to request *a priori* a distribution of funds to match the optimal requirement.

3.4 Time Dependencies Among Decisions

Operational considerations give rise to coercion sets that ensure continuity of mission availability between time periods, in addition to pairwise interactions that can be applied to one or more time periods. For example (Figure 5) a coercion subset may be used to denote a weapon system w (or collection of weapon systems), with its corresponding start- and stop-service years, $\underline{y}$ and $\overline{y}$, whose operation is dependent upon another weapon system w', with its own corresponding start- and stop-years, $\underline{y}'$ and $\overline{y}'$.

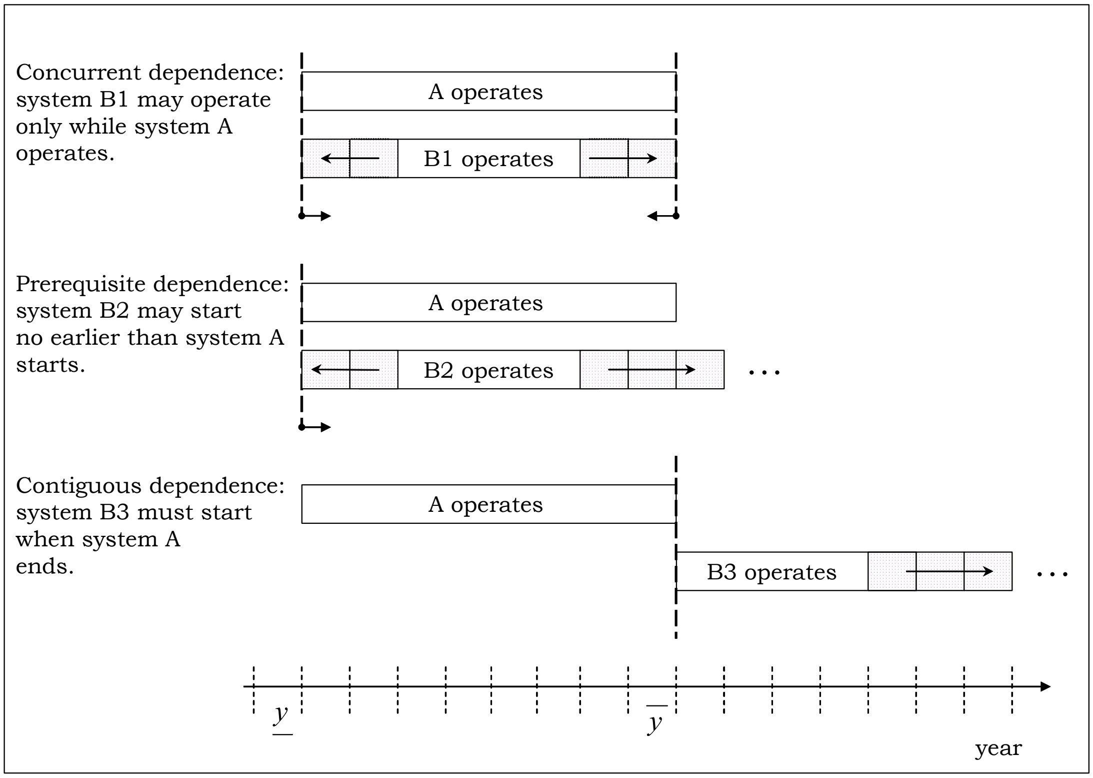

Figure 5: When we begin operating a new system can depend on the timing of the other systems' operations. System A starts operating at the end of year $\underline{y}$ and stops at the end of $\overline{y}$. The concurrent dependence of candidate system B1 on A restricts it to operating only when A does. The prerequisite dependence of B2 on A prevents it from starting before A does. The contiguous dependence of B3 on A requires it to start operating right after A ceases operation.

Some weapon systems must be procured so that their service years are synchronized in some way with those of other weapon systems. Weapon system *w*, procured under acquisition option *a*, may be required to operate concurrently with weapon system w' procured under acquisition option a', provided $\underline{y} \le \underline{y}' \wedge \overline{y} \ge \overline{y}'$. We term this "concurrent operation," and impose the constraint:

$$SELECT_{a'} \le SELECT_{a}.$$

Weapon system *w*, procured under one of the acquisition options $a \in \Omega$, may be required to operate prior to weapon system w' procured under acquisition option a', provided $\overline{y} \le \underline{y}'$. We term this "prerequisite operation," and impose the constraint:

$$SELECT_{a'} \le \sum_{a \in \Omega} SELECT_{a}.$$

Weapon system *w*, procured under exactly one of the acquisition options $a \in \Omega$, may be required to operate immediately after weapon system w' procured under acquisition option a', provided $\underline{y}' = \overline{y} + 1$. We term this "contiguous operation," and impose the constraint:

$$SELECT_{a'} = \sum_{a \in \Omega} SELECT_a .$$

3.5 Mechanisms to Regulate Long-term Capital Plans

We have shown how budgets may be specified in funding categories and fiscal years. Funding categories may be yearly and strictly partitioned, *e.g*., Field [1999], while other funding restrictions may span some multiple-year epoch, *e.g*., space systems funding allocated to five-year epochs [Newman *et al*. 2000]. Some funding categories are fungible with others, resulting in planning models with some amalgam of separate categories and blended funds. For example, Brown, Clemence, Teufert, and Wood [1991] mingle Army helicopter construction costs and a host of ancillary expenses. Each military service has its own signature planning regimen. Builder [1987] lampoons these distinctions with painful accuracy, great humor, and good taste.

Because the costs of selecting acquisition options relative to budgeted funding categories are high, it can be very difficult to select a portfolio such that the resulting annual outlays fit exactly in each funding category in each planning year of the horizon. In other words, a superficially simple annual budget constraint over a long planning horizon is ridiculous in the real world.

To address this recurring problem, some planning models employ a *budget band* over the planning horizon, with yearly lower and upper bounds on each funding category in the short term, and larger bands farther into the future to reflect planning uncertainties. The use of these bands provides some reasonable degree of freedom as to when funds are spent, even if the total amount spent does not change.

Even with budget bands, we can encounter an optimal solution that is silly, such as leaving a large amount of money unused in some category and year, even though with just a few dollars more we could select an attractive alternative. To avoid such foolishness, we create an *elastic constraint* on the budget. Rather than overlook some solution that is almost, but not quite, feasible, we allow a budget band to be violated, albeit at a high elastic penalty cost per unit of violation. An insightful solution with small, cosmetic elastic violations is much better received by planners than a strictly feasible solution that nobody likes.

With elastic yearly budget bands, we can still encounter an optimal solution with myopic elastic violations over the planning years: violations that, after some analysis, can be shifted sooner and/or later to reduce their number and/or severity. To capture this in the optimization model, we employ *cumulative elastic constraints*, where each (*e.g.*, upper) limit each year is replaced by the sum of all such limits from the first year, up to and including that year. Here, an elastic violation in any year keeps reappearing in later years, unless it is offset by some compensating later event.

After a plan has been refined, and, perhaps, promulgated to senior leadership, changes can arise that necessitate revision of this legacy. Optimization has a well-earned reputation for amplifying small changes to input parameters into breathtaking changes to output plans. Often, disruptive changes turn out to be unnecessary, exhibiting total costs varying insignificantly from the legacy plan. Brown, Dell, and Wood [1997] show how to incorporate *persistence* in optimization-based decision support models.

For example, suppose we have a binary legacy solution $SELECT_a$* against which we would like to compare the solution $SELECT_a$. The linear expression:

$$\sum_{a|SELECT_a*=0} SELECT_a + \sum_{a|SELECT_a*=1} (1 - SELECT_a)$$

sums the number of revisions (the "Hamming distance") between the legacy solution and the revision, which can be constrained and/or elastically penalized. Restricting the differences between the two solutions, *i.e.*, providing an upper bound on the expression above, will not reduce costs, but frequently reveals alternate solutions that cost little more and exhibit much less turbulence.

Additionally, we can use a simplifying restriction of Hamming distance to generate a family of high-quality solutions as follows. Find an optimal plan. Report it, and then add a constraint to preclude just this plan in its entirety, or as a subset, *e.g.*,

$$\sum_{a|SELECT_a*=1} SELECT_a \leq \left(\sum_a SELECT_a^* \right) - 1.$$

Find another optimal plan, report it, similarly restrict it from further consideration, and continue.

Given that these planning problems can be large linear integer programs, each of which is difficult to solve at all, let alone exactly, we assume that some interval of uncertainty, or integrality tolerance, will reasonably apply. If, *e.g.*, each plan is "within 5 percent of optimality," then the successively-restricted plans will not necessarily exhibit monotonically nondecreasing costs.

Long-term capital planning models have finite planning horizons. Beginning and ending states are required as input parameters. When the purpose of long-term planning is to advise how to evolve, it seems paradoxical that the end state must be specified as an input, rather than learned as an output. Worse, the end state is a long time from now and not easy to reckon. Errors and omissions in determining end states can lead to *end effects*: outrageous behavior at the end of the planning horizon.

End effects can arise from simple causes. For instance, an innocent requirement that "no purchased alternative can be retired before it has been in service 10 years," combined with the requirement that "every purchased alternative must be retired before the end of the planning horizon" can preclude any procurement in the last planning decade. Often, the causes of end effects are much more subtle, and thus the magnitude and duration of their impact can be exceedingly difficult to gauge. Although the end effects problem has been examined theoretically, *e.g.*, Walker [1995], results to date are limited to special cases. The key, theoretically or heuristically, is to induce end states that do not unduly coerce the results that lead there.

Common sense prevails when addressing end effects. One way to mitigate end effects is to extend the planning horizon beyond those years actually reported. (Many of our references do this.) The problem here (both theoretical and practical) is to determine just how long the time horizon should be extended. In the trivial case above, at least a decade extension is advisable.

In any real-world, long-term planning model, there are allowances for bad events; unavoidable despite optimization. Given a choice, we prefer to delay such bad news as long as possible into the future by discounting the penalty for such an event at a higher rate than its companion costs in the model. We call this the "fog of far-future planning factor," or the *model mischief discount rate*.

3.6 When Objectives are Constraints, and Vice Versa: Dealing With Multiple, Conflicting Measures of Effectiveness

Military capital planning models naturally attract guidance from senior leadership. Respecting the source of such advice, it is natural to want to add constraints restricting the model to follow such guidance verbatim. However, expert opinions can be inconsistent between sources, and the intersection of all the guidance provided almost certainly renders a model infeasible, offering a mathematical example of the old adage "you can't please everyone."

A classic method for simultaneously addressing multiple objectives is simple, but often does not produce a globally-accepted solution: use some weighted-average of all objectives (*e.g.*, Steuer [1986], Chapter 7). This weighted-average objective must have some supernumerary units, requiring that we somehow equilibrate "apples plus oranges plus grenades plus . . ."

A weighted average can be used to try to coerce hierarchy among component objectives; assuming, of course, that we can establish such a well-ordered hierarchy. However, this is problematic even for just two objective components, if not complete *adhocery*. Textbook descriptions of the "Big-M" multiplier method to achieve a feasible solution suffice to illustrate. Just how big does "Big-M," the objective weight per unit of constraint infeasibility, have to be to guarantee a hierarchical distinction? Our military world record is a model sent to us with 14 hierarchical objectives: even if each objective is given a weight just one order of magnitude higher than the next lower objective, the resulting weighted-average objective exceeds the mantissa length of an IEEE 80-bit floating point number, so (even without a course in numerical analysis) one can see that this objective is inflicted with worrisome, if not overwhelming, rounding-error noise.

We can achieve purely hierarchical solutions without weighted averages (*e.g.*, Steuer [1986], Chapter 9). First, optimize only with the highest-order objective, and then write an *aspiration constraint* requiring at least the resulting optimal value of this objective function. Add the aspiration constraint to the existing set of constraints and reoptimize, this time with the second-highest order objective. Repeat this with each successive lower-level objective.

Pursuing strict hierarchies among conflicting objectives and/or conflicted guidance can obscure good trade-offs. This leads us to relax our aspiration constraint for each objective to an *elastic aspiration constraint*, where we express some goal level of achievement and, as before, allow violation with a linear elastic penalty.

Sometimes, we are only given a list of MOEs, and it is left to us to determine what the aspiration levels should be. This leads to a series of optimization problems in which each requirement may take its turn in the objective, while its cohorts play the role of elastic aspiration constraints. That is, we are empirically discovering MOE levels that admit efficient solutions. A common heuristic is to cycle through the MOEs in some priority order, finding the extremal (*i.e.*, maximizing or minimizing) value of each, setting some fraction of this as its aspiration, and continuing to the next MOE.

4. Computational Examples and Guidance

Military capital planning problems typically address numerous assets, (*e.g.*, weapon systems, munitions, platforms, and vehicles), many years in the planning horizon, and many acquisition options—limited only by the imagination of procurement planners and the responsiveness of competing contractors. As a result, optimization-based decision support models of military capital planning problems are large and complex, typically resulting in many thousands of discrete and continuous variables and thousands of constraints. Not all such models can be solved quickly. In fact, a model is tractable only if it can be relied on to render a useful answer while we still remember the question. But, important problems deserve serious analysis. We military analysts have lots of computing power. The authors think nothing of solving hundreds, or thousands, of planning scenarios. When the objective is billions of taxpayer dollars and the result is a durable decision to invest in the future defense of our country, analysis is everything.

We present a few tricks that usually make large models easier to solve. To motivate, the next two paragraphs each report an anecdotal application and evidence of where we hope to be when we finish crafting, tuning, and tweaking.

The Air Force uses a capital-planning model [Newman *et al*. 2000] that features about 10,000 variables and about 17,000 constraints; between 2% and 10% of the decision variables are required to be binary. Answers within 2% of optimality are produced in about three minutes on a Silicon Graphics ONYX2 workstation with 4 gigabytes of RAM using the CPLEX solver, Version 6.5 [ILOG 2002].

A capital-planning model designed for the Navy to plan the procurement of Naval ships and aircraft for the next 25 years [*e.g.*, Garcia 2001] has about 167,000 variables (about 6,000 binary), and about 114,000 constraints. Heuristic solutions are generated in a second or two, and solutions within 10% of optimality are produced in about seven minutes on a 1GHz Pentium III computer with 1 gigabyte of RAM using the CPLEX Solver, Version 6.5.

4.1 Time Discount Rate and Model Mischief Discount Rate

We use present value for all costs, but we also use a model mischief discount rate to attenuate the influence of far-future constraints when these constraints cause trouble. This reflects planning reality, and can also help the optimization algorithm distinguish between admissible solutions.

Discount rates help Newman *et al.* [2000] with capital-planning for the Air Force Space Command. Base-case, nondiscounted model instances require about 1½ hours to solve to an optimality gap of 10%. However, applying a 2.5% annual discount factor reduces solution times for these same instances to between 6 minutes and 1 hour to solve to the same 10% optimality gap. Analysis of the discounted model solutions reveals no degradation in solution quality.

Salmeron, Brown, Dell, and Rowe [2002] express all naval procurement, operation and maintenance costs in constant-year dollars, but add an extra “inflation” factor to realistically represent operation and maintenance costs for older aircraft. The decrease in solution effort here proves so dramatic that nondiscounted base cases are not attempted. These authors also allow violations of cumulative budget bands, and use a model mischief discount rate to move any such violations as far as possible into the future. This proves worthwhile, because their model also exhibits end effects. For example, near the end of the long planning horizon, ships are forced to retire without alternate replacements on the drawing boards. Here, it is better to deliver workable advice with excellent near-term fidelity than to let this far-term blemish shatter the entire planning exercise.

4.2 Relaxation and Aggregation

A decision to select an acquisition option in a particular year may have to be binary in the near term, but not in the far term. Integer acquisition quantities are required, especially in the near term, if the magnitudes are small, but we frequently relax integer decision requirements in the intermediate and far terms, which can have a dramatic effect on model performance.

For example, we might replace binary alternatives to select some acquisition option in exactly one out of a set of future years with a relaxation that permits the alternative to be fractionally selected during that epoch, but fully selected by the end of it. This permits planned investments to be spread over years in the future, *e.g.*, Newman *et al.* [2000], rather than forcing the investments to be made in some particular year. Salmeron, Brown, Dell, and Rowe [2002] use continuous quantities for aircraft and all retirements, and these account for the vast majority of the nearly 100,000 decision variables. Some ship quantities may be continuous in the far future, permitting planned construction to span planning years.

These relaxations are not always easy to express, say, when there are interactions among far-term decisions. Aggregation can help limit model sizes and hasten planning cycles, while reducing the workload of preparing parameters for far-future alternatives. In the near term, we might want a diversity of alternatives, while in the far term, we might just have one alternative to select, or reject. In the near term, we might need yearly time fidelity, while in the far term, we can aggregate yearly constraints and variables to model an entire decade, reflecting the reality that timing selections in the out-years are less precise. When the planning horizon rolls forward, all model features will eventually be disaggregated and amplified to then-present value.

4.3 Strong Formulation

Recall the coercion "this option must be selected to enable any option in that set to be selectable." If we refer to "this option" as the controlling binary variable $SELECT_{a'}$, and to "that set" as the controlled variables in the set Ω, this may be formulated as a single linear constraint:

$$\sum_{a \in \Omega} SELECT_a \leq |\Omega| SELECT_{a'}.$$

Alternately, we can use $|\Omega|$ linear constraints:

$$SELECT_a \leq SELECT_{a'} \quad \forall a \in \Omega.$$

Even though this latter constraint set is superficially equivalent to the former constraint for binary selections, the latter set is much stronger than the former for continuous relaxations of selections (*e.g.*, Rardin [1998], Chapter 12), and thus a much better guide for enumeration. There are many more examples of alternate formulations of particular model features, but the principles are the same: try to tighten the continuous relaxation as much as possible, without encumbering the model with an overwhelming number of constraints.

4.4 Setting Elastic Penalties

The *manager's principle* says "only pay attention to taut constraints, and only worry about critical resources." This is an *a posteriori* policy: first, produce a plan; then see what trouble you need to attend to. Elastic constraints generalize this concept to say, *a priori*, "if you violate this constraint, here is how much it will cost, per unit violated." Using elastic constraints allows any plan you produce to follow your advice on when and how to violate constraints.

Elastic penalties are useful, but setting them deserves some care and analysis. If violations arise (and in military capital planning models, violations are inevitable) the objective function expresses the impact of these violations, and the resulting planning solution will be influenced. Each elastic penalty is a bound on the linear program dual variable of its constraint. If you set penalties reasonably, you will be rewarded with a tractable model and plans that have followed your considered advice. If you set penalties carelessly, you will end up wishing you hadn't used elastic constraints at all.

5. Conclusion

The military has employed mathematical optimization of capital planning for over 50 years and has made many contributions to this field. Planning capital outlays for the military is different than for the private sector, principally in the sheer amounts of money involved, the nature of the procurement costs, and additional requirements and regulations imposed on military capital expenditures. This report distills many recurrent planning issues and shows how they can be expressed in a mathematical model. Many of these ideas are not contained in any textbook and we recommend the references in our bibliography for more detail.

Acknowledgements

The authors acknowledge Professors Kevin Wood of the Naval Postgraduate School and J. K. Newman of the University of Illinois for their helpful comments and suggestions, and thank Janel Brooks for retrieving some background material.

REFERENCES

Ashton, D. and D. Atkins. [1979]. "Rules of Thumb and the Impact of Debt in Capital Budgeting Models," *Journal of the Operational Research Society*, Vol. 30, No. 1, pp. 55-61.

Bailey, R. [1953]. "Application of Operations-Research Techniques to Airborne Weapon System Planning," *Journal of the Operations Research Society of America*, Vol. 1, No. 4, pp. 187-199.

Baker, S., D. Morton, L. Williams, and R. Rosenthal. [2002]. "Optimizing Strategic Airlift," *Operations Research*, Vol. 50, No. 4, pp. 582-602.

Baran, N. [2000]. "Optimizing Procurement Planning of Navy Ships and Aircraft," MS Thesis in Operations Research, Naval Postgraduate School, Monterey, CA, December.

Baumol, W. and R. Quandt. [1965]. "Investment and Discount Rates Under Capital Rationing—A Programming Approach," *Economic Journal*, Vol. 75, pp. 317-329.

Bayram, V. [2002]. "Optimizing the Capacity and Operation of U.S. Army Ammunition Production Facilities," MS Thesis in Operations Research, Naval Postgraduate School, Monterey, CA, June.

Bernhard, R. [1969]. "Mathematical Programming Models for Capital Budgeting—A Survey, Generalization, and Critique," *Journal of Financial and Quantitative Analysis*, Vol. 4, No. 2, pp. 111-158.

Bertsimas, D. and J. Tsitsiklis. [1997]. *Introduction to Linear Optimization*, Athena Scientific, Belmont, MA.

Bhaskar K. [1974]. "Borrowing and Lending in a Mathematical Programming Model of Capital Budgeting," *Journal of Business Finance & Accounting*, Vol. 1, pp. 267-291.

Bradley, G. [1986]. "Optimization of Capital Portfolios," *Seminar OPS-01*, *National Communications Forum*, Chicago, September, pp. 11-17.

Bradley, G. [2003]. "GTE Capital Portfolio Optimization Documentation," private communication, August.

Bradley, S. and S. Frey, Jr. [1978]. "Equivalent Mathematical Programming Models of Pure Capital Rationing," *Journal of Financial and Quantitative Analysis*, June, pp. 345-361.

Brown, G., R. Clemence, W. Teufert, and R. Wood. [1991]. "An Optimization Model for Modernizing the Army's Helicopter Fleet," *Interfaces*, Vol. 12, No. 4, pp. 39-52.

Brown, G., D. Coulter, and A. Washburn. [1994]. "Sortie Optimization and Munitions Planning," *Military Operations Research*, Vol. 1, No. 1, pp. 13-18.

Brown, G., R. Dell, and R. Wood. [1997]. "Optimization and Persistence," *Interfaces*, Vol. 27, No. 5, pp. 15-37.

Brown, G., R. Dell, H. Holtz, and A. Newman. [2003]. "How U.S. Air Force Space Command Optimizes Long-Term Investment in Space Systems," *Interfaces*, Vol. 33, No. 4, pp. 1-14.

Broyles, J. [1976]. "Compact Formulations of Mathematical Programmes for Financial Planning Problems," *Operational Research Quarterly*, Vol. 24, No. 4, pp. 885-893.

Bruggeman, J. [2003]. "A Multi-Year Ammunition Procurement Model for Department of the Navy Non-Nuclear Ordnance," MS Thesis in Operations Research, Naval Postgraduate School, Monterey, CA, September.

Buffum, R. [1978]. "Workforce Planning Models for the Naval Air Test Center," MS Thesis in Operations Research, Naval Postgraduate School, Monterey, CA, September.

Builder, C. [1987]. "The Army in the Strategic Planning Process: Who Shall Bell the Cat?," RAND Report R-3513, Santa Monica, CA, April.

Carleton, W. [1969]. "Linear Programming and Capital Budgeting Models: A New Interpretation," *Journal of Finance*, Vol. 24, pp. 825-833.

Chambers, J. (ed.). [1999]. *American Military History*, Oxford University Press, Oxford, United Kingdom.

Dantzig, G. [1963]. "Linear Programming and Extensions," Princeton University Press, Princeton, NJ.

Dobson, G. [1982] "Worst-Case Analysis of Greedy Heuristics for Integer Programming With Nonnegative Data," *Mathematics of Operations Research*, Vol. 7, No. 4, pp. 515-531.

Edwards, T. [1986]. "Capital Portfolio Optimization: Results," *Seminar OPS-01*, *National Communications Forum*, Chicago, September, pp. 17-18.

Everett, H. [1963]. "Generalized Lagrange Multiplier Method for Solving Problems of Optimal Allocation of Resources," *Operations Research*, Vol. 11, No. 3, pp. 399-417.

Evans, G. and R. Fairbairn. [1989]. "Selection and Scheduling of Advanced Missions for NASA Using 0-1 Integer Linear Programming," *Journal of the Operational Research Society*, Vol. 40, No. 11, pp. 971-981.

Field, R. [1999]. "Planning Capital Investments in Navy Forces," MS Thesis in Operations Research, Naval Postgraduate School, Monterey, CA, December.

Fox, G., N. Baker, and J. Bryant. [1984]. "Economic Models for R and D Project Selection in the Presence of Project Interactions," *Management Science*, Vol. 30, pp. 890-902.

Garcia, R. [2001]. "Optimized Procurement and Retirement Planning of Navy Ships and Aircraft," MS Thesis in Operations Research, Naval Postgraduate School, Monterey, CA, December.

Geoffrion, A. [1986]. "Capital Portfolio Optimization: A Managerial Overview," *Seminar OPS-01, National Communications Forum*, Chicago, September, pp. 6-10.

Glomski, R. [1986]. "Evolving Capital Program Management Process," *Seminar OPS-01*, *National Communications Forum*, Chicago, September, pp. 2-5.

Gunther, P. [1955]. "Use of Linear Programming in Capital Budgeting," *Journal of the Operations Research Society of America*, Vol. 3, No. 2, pp. 219-224.

ILOG. [2002]. “CPLEX 6.5,” [http://www.cplex.com], accessed July.

Kaplan, S. [1966]. “Solution of the Lorie-Savage and Similar Integer Programming Problems by the Generalized Lagrange Multiplier Method,” *Operations Research*, Vol. 4, pp. 1130-1136.

Karabakal, N., J. Lohmann, and J. Bean. [1994]. “Parallel Replacement under Capital Rationing Constraints,” *Management Science*, Vol. 40, No. 3, pp. 305-319.

Keeney, R. [1992]. *Value-Focused Thinking*, Harvard University Press, Cambridge, MA.

Keown, A. and J. Martin. [1976]. “An Integer Goal Program for Capital Budgeting in Hospitals,” *Journal of the Operational Research Society*, Vol. 34, No. 5, pp. 28-35.

Keown, A. and B. Taylor III. [1980]. “A Chance Constrained Integer Goal Programming Model for Capital Budgeting in the Production Area,” *Journal of the Operational Research Society*, Vol. 31, No. 7, pp. 579-589.

Kimms, A. [2001]. *Mathematical Programming and Financial Objectives for Scheduling Projects*, Kluwer Academic Publishers, Norwell, MA.

Koopmans, T. [1951]. “Activity Analysis of Production and Allocation,” Proceedings of a conference edited by T.C. Koopmans, John Wiley & Sons, NY.

Kuhn, H and A. Tucker. [1950]. “Non-Linear Programming,” *Proceedings of the Second Symposium on Mathematical Statistics and Probability*, Berkeley, CA.

Kumar, P. and T. Lu. [1991]. “Capital Budgeting Decisions in Large Scale, Integrated Projects: Case Study of a Mathematical Programming Application,” *The Engineering Economist*, Vol. 36, No. 2, pp. 127-150.

Loerch, A. [1999]. “Incorporating Learning Curve Costs in Acquisition Strategy Optimization,” *Naval Research Logistics*, Vol. 46, No. 3, pp. 255-271.

Loerch, A., R. Koury, and D. Maxwell. [1999]. “Value Added Analysis for Army Equipment Modernization,” *Naval Research Logistics*, Vol. 46, No. 3, pp. 233-253.

Lofti, V., J. Sarkis, and J. Semple. [1998]. “Economic Justification for Incremental Implementation of Advanced Manufacturing Systems,” *Journal of the Operational Research Society*, Vol. 49, No. 8, pp. 829-839.

Lorie, J. and L. Savage. [1955]. “Three Problems in Rationing Capital,” *The Journal of Business*, Vol. 28, No. 4, pp. 229-239.

Lusztig, P. and B. Schwab. [1968]. “A Note on the Application of Linear Programming to Capital Budgeting,” Journal of Financial and Quantitative Analysis, Vol. 3, No. 4, pp. 427-431.

Mamer, J. and A. Shogan. [1987]. “A Constrained Capital Budgeting Problem with Applications to Repair Kit Selection,” *Management Science*, Vol. 33, No. 6, pp. 800-806.

Marschak, J. and M. Mickey. [1954]. “Optimal Weapon Systems,” *Naval Research Logistics Quarterly*, Vol. 1, No. 2, pp. 116-140.

McCullough, D. [2001]. *John Adams*, Simon and Schuster, New York, NY.

Meier, H., N. Christofides, and G. Salkin. [2001]. "Capital Budgeting Under Uncertainty—An Integrated Approach Using Contingent Claims Analysis and Integer Programming," *Operations Research*, Vol. 49, No. 2, pp. 196-206.

Morton, D. [2003]. "Optimizing Strategic Airlift," private communication, August.

Naval Center for Cost Analysis. [2002]. [http://www.ncca.navy.mil], accessed July.

Newman, A., G. Brown, R. Dell, A. Giddings, and R. Rosenthal. [2000]. "An Integer Linear Program to Plan Procurement and Deployment of Space and Missile Assets," NPS Technical Report NPS-OR-00-005, Naval Postgraduate School, Monterey, CA.

Parnell, G., H. Conley, J. Jackson, L. Lehmkuhl, and J. Andrew. [1998]. "Foundations 2025: A Value Model for Evaluating Future Air and Space Forces," *Management Science*, Vol. 44, No. 10, pp. 1336-1350.

Pasztor, A. [1990]. "Navy's A-12 Attack Jet Program is Said to be Facing a $4 Billion Cost Overrun," *The Wall Street Journal*, December 10.

Pasztor, A. [1991]. "Navy Head, Reversing Himself, Concedes He Got Early Word on A-12 Cost Overrun," *The Wall Street Journal*, April 9.

Pfarrer, M. [2000]. "Optimizing Procurement of Special Operations Weapons and Equipment," MS thesis in Operations Research, Naval Postgraduate School, Monterey, CA, June.

Rardin, R. [1998]. *Optimization in Operations Research*, Prentice Hall, Upper Saddle River, NJ.

Ricks, T. [1999]. "Navy to Stay with Joint Fighter Despite Cost Overrun at Lockheed Martin," *The Wall Street Journal*, February 26.

Reuters. [1985]. "Aerojet Contract," *The New York Times*, August 23.

Rychel, D. [1977]. "Capital Budgeting with Mixed Integer Linear Programming: An Application," *Financial Management*, Winter, pp. 11-19.

Salmeron, J., G. Brown, R. Dell, and A. Rowe. [2002]. "An Optimization-Based Decision-Support System to Plan Procurement and Retirement of Naval Platforms," NPS Technical Report NPS-OR-02-006, Naval Postgraduate School, Monterey, CA.

Sandler, T. and K. Hartley. [1995]. *The Economics of Defense*, Cambridge University Press, Cambridge, United Kingdom.

Senju, S. and Y. Toyoda. [1968]. "An Approach to Linear Programming with 0-1 Variables," *Management Science*, Vol. 15, No. 4, pp. B196-B207.

Stanley, E., D. Honig, and L. Gainen. [1954]. "Linear Programming in Bid Evaluation," *Naval Research Logistics Quarterly*, Vol. 1, No. 1, March, pp. 48-54.

Steuer, R. [1986]. "Multiple Criteria Optimization: Theory, Computation, and Application," Wiley, New York, NY.

Sweet, T. [1986]. "Optimizing the Capital Investment Portfolio," *Seminar OPS-01*, *National Communications Forum*, Chicago, September, p. 2.

Taylor, B. III, A. Keown, and A. Greenwood. [1983]. “An Integer Goal Programming Model for Determining Military Aircraft Expenditures,” *Journal of the Operational Research Society*, Vol. 34, No. 5, pp. 379-390.

Toyoda, Y. [1975]. “A Simplified Algorithm for Obtaining Approximate Solutions to Zero-one Programming Problems,” *Management Science*, Vol. 21, No. 12, pp. 1417-1427.

Unger, V. [1970]. “Capital Budgeting and Mixed Zero-One Integer Programming,” *AIIE Transactions*, Vol. 2, pp. 28-36.

Walker, S. [1995]. “Evaluating End Effects for Linear and Integer Programs Using Infinite-Horizon Linear Programming,” Ph.D. Dissertation in Operations Research, Naval Postgraduate School, Monterey, CA, March.

Weingartner, H. [1966]. “Capital Budgeting of Interrelated Projects: Survey and Synthesis,” *Management Science*, Vol. 12, No. 7, pp. 485-516.

Weingartner, H. [1977]. “Capital Rationing: *n* Authors in Search of a Plot,” *Journal of Finance*, Vol. 32, pp. 1403-1431.

Yost, K. [1996]. “The Time Strike Optimization Model,” NPS Technical Report, NPS-OR-96-001, Naval Postgraduate School, Monterey, CA, January.

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